THE INSTITUTE FOR FINANCIAL MARKETS

Portfolio Theory & Equilibrium Models For Interactive Brokers, LLC

Kevin Baldwin

THE INSTITUTE FOR FINANCIAL MARKETS

The risk of loss in trading commodities can be substantial. You should therefore carefully consider whether such trading is suitable for you in light of your financial condition.

The high degree of leverage that is often obtainable in commodity trading can work against you as well as for you. The use of leverage can lead to large losses as well as gains.

The information contained herein is derived from sources believed to be reliable. The audience should practice their own due diligence.

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Kevin Baldwin

Kevin Baldwin began his career in the futures and options industry 16 years ago with Refco's Institutional Applications department. He taught Refco's six week futures and options course in Chicago for six years. In addition to the six week Chicago program, he provided shorter term derivatives seminars for client institutions in Buenos Aires, Rio de Janeiro, Tokyo, Seoul, and Moscow on behalf of the British government's Know How fund. Mr. Baldwin also was an instructor for the Illinois Institute of Technology's Master's Program in Financial Markets.

In 1996, Kevin Baldwin joined one of Refco's innovative Introducing Broker's in New York City as managing director and had held various securities and futures registrations including Series 3, 4, 7, 24, 30 and 63. In addition to his professional responsibilities, Kevin became an adjunct faculty member for New York University's School of Continuing Education where he taught both Intermediate Securities Analysis and Futures and Options courses. In 2000, Kevin returned to Chicago and developed a portfolio of websites aimed at different segments of the futures and options community. More recently, he has offered an updated variety of in-house 3-day to 3-week futures and options courses to firms.

Mr. Baldwin earned a bachelor of science degree from San Jose State University in California, and an MBA from the University of Chicago, Graduate School of Business.

Outline

- Efficient Markets
- Market Model
- σ_i , $\sigma_{portfolio}$ and Total risk
- Covariance and Correlation
- Efficient portfolio's, CML, SML
- CAPM (Sharpe & Lintner)
- Performance Indices

Efficient Markets

'Efficient markets' are efficient in processing information. AND , security prices are based on the correct evaluation of all information.

Mean Variant Efficient assets are those assets with the highest expected return for a given amount of risk.

Perfectly Efficient

1. Information is free
2. No transaction cost or tax
3. Investors are price takers
4. Investors seek maximum utility

Economically Efficient

1. Infomation is free
2. Investors are price takers
3. Investors seek maximum utility.

Maximizing utility is minimizing σ (risk) while maximizing E(ret.)

Efficient Markets as ‘hypothesis’

★ The hypothesis states that all information is reflected in security prices.

DUAL TEST

- That the Model is correctly specified?
- Is the Market efficient?

Forms of Market Efficiency

- *Weak Form*- You cannot use historical information to generate abnormal profits.
- *Semi-Strong Form*- Currently available information cannot be used to generate abnormal profits.
- *Strong Form*- All information (public & private) is discounted into security prices.

The <u>Market Model</u> says that security prices behave as a (linear) function of the overall market.

Let R_i = return on stock_i ,

R_m = return on the market (S&P 500)

$$E(R_i) = \alpha_i + \hat{\beta}_i R_m + \varepsilon_i$$

Which Form?

Relationships between variables may be explained using a variety of techniques using various assumptions. The first assumption is the *form* of the relationship:

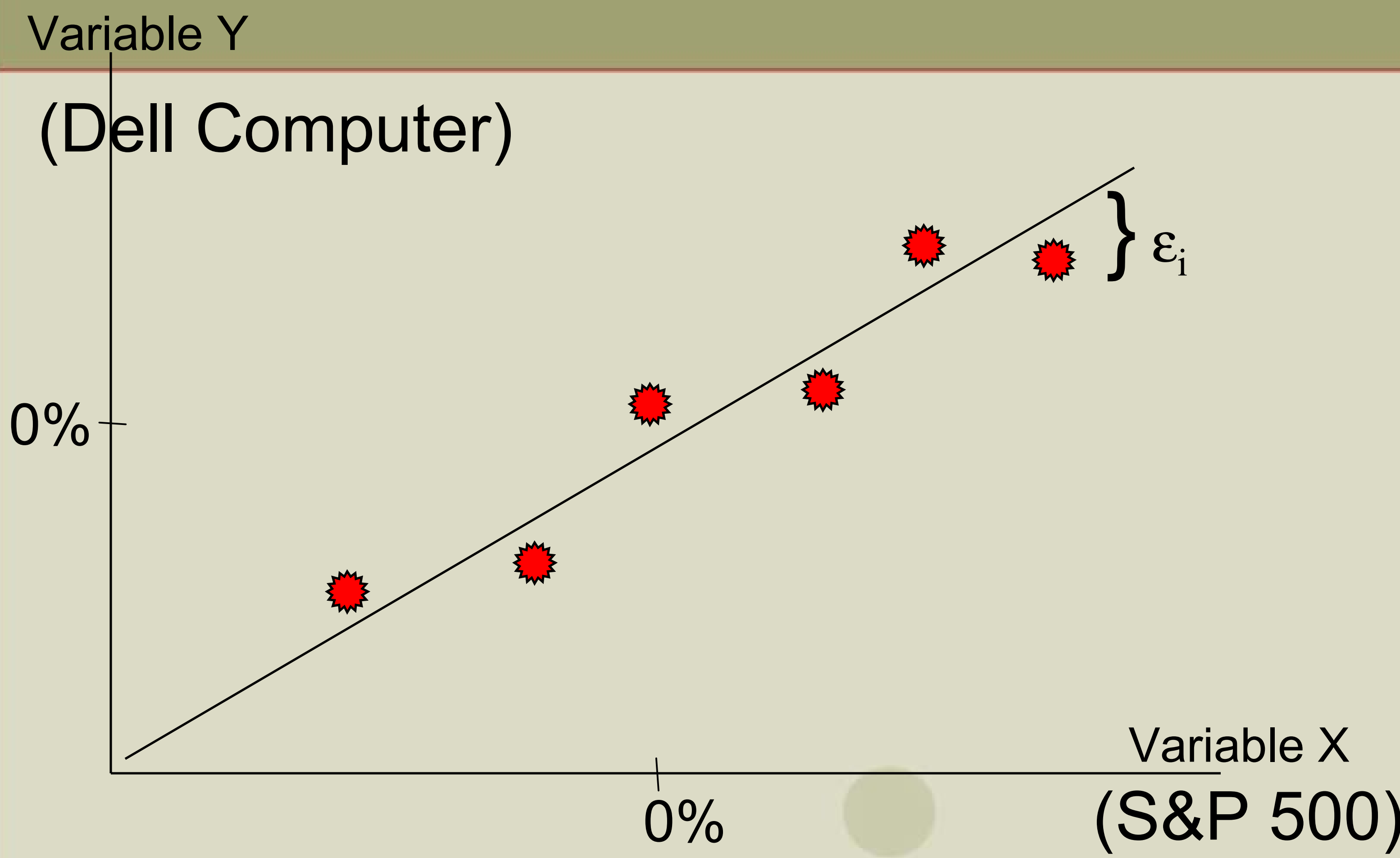
Linear: $R_i = \alpha_i + \beta_i R_m + \varepsilon_i$

Exponential: $R_i = \alpha_i \exp(\beta_i R_m)\, \varepsilon_i$

Logistic: $R_i = \dfrac{\alpha_i}{1 + \beta_i \exp(\beta_2 R_m)}\, \varepsilon_i$

Gauss Markov Linear Regression

- Dependent Variable R_i varies as a linear function of the independent variable R_m.
- Minimize the squared deviations from the mean.
- Residuals $\varepsilon_i = (R_i - \hat{R}_i)$, and $E(\Sigma \varepsilon_i^2) = 0.0$
- $\hat{\beta}_i$ is really an estimate of β_i thus it has a probability density.
- The least squares estimators, in the class of unbiased linear estimators, have minimum variance, that is, they are 'BLUE'.



Linear: $y_i = \hat{\alpha}_i + \hat{\beta}_i x_i + e_i$

thus, if $\beta = 1.7$ and $\alpha = 0$, then the solution to the function when x =0.10 is 0.17 ; this is the prediction of y_i when x =.1

HISTORICAL BETA

DELL US Equity

Relative Index SPX

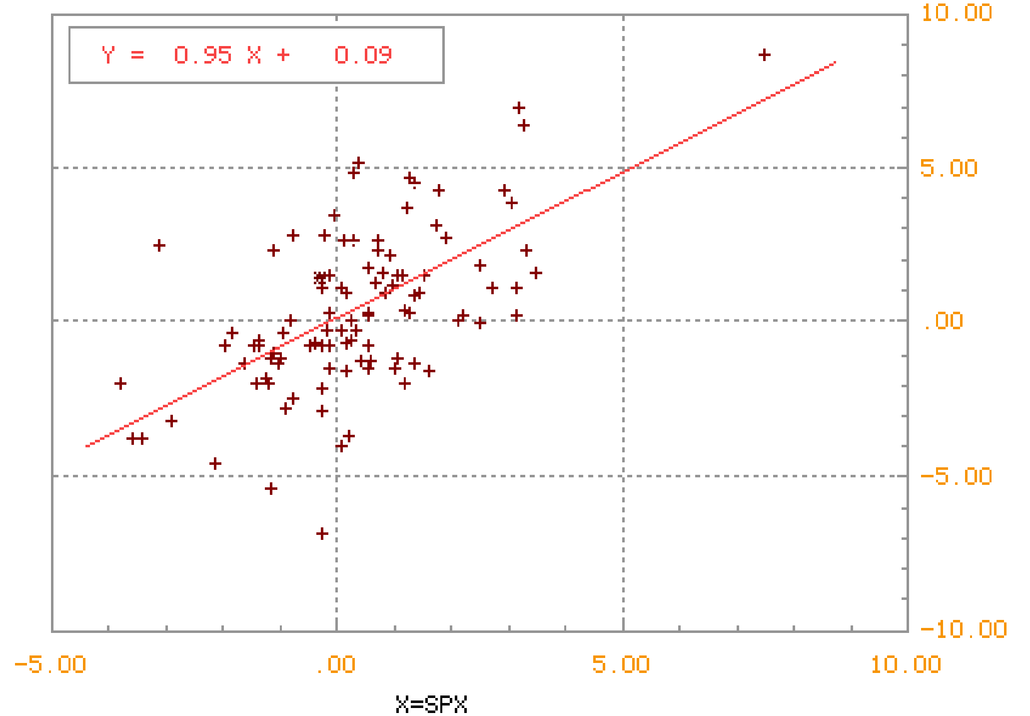
Period W Weekly
Range 2/28/03 To 2/18/05
Market T Trade

ADJ BETA	0.96
RAW BETA	0.95
Alpha(Intercept)	0.09
R2 (Correlation)	0.35
Std Dev of Error	2.21
Std Error of Beta	0.13
Number of Points	103

ADJ BETA = (0.67) * RAW BETA
+ (0.33) * 1.0

DELL INC

S&P 500 INDEX
*Identifies latest observation



Where: Adj.Beta = 0.67(Raw Beta) + 0.33(1.0)

Linear Variation explained by regression

$$r^2 = \frac{ESS}{TSS} = \frac{\Sigma(\hat{R}_i - \bar{R})^2}{\Sigma(R_i - \bar{R})^2}$$

ESS = Explained Sum of Squares
TSS = Total Sum of Squares

Where:

$$\hat{R}_i = \hat{\alpha}_i + \hat{\beta}_i R_m \quad = \text{'Fitted Value'}$$

Coefficient of Determination = r^2

r^2 is the amount of variation that is explained by the regression.

$\sqrt{r^2} = r$ is the coefficient of correlation; which is routinely interpreted in terms of its squared value.

Many text use the notation:
SS(Total) = SS (Regression) + SS(Residuals)

$$r^2 = \frac{SS(\text{total}) - SS(\text{residuals})}{SS(\text{total})}$$

SS=sum of squares

Properties of r

1. It is a measure of linear association or linear dependence only.
2. It can be positive or negative between the bounds $-1.0 < r < +1.0$
3. If X and Y are statistically independent they will have an $r=0$. Thus, when $r = 0$; x does not help in predicting y. HOWEVER, an $r = 0$ does not imply independence.

An Example

Measure the strength of linear association between GE and SPX (S&P500 cash) during the period Feb 15,2004 to Feb 15, 2005.

Regression Analysis

The regression equation is

GE = - 0.113 + 1.11 SPX

Predictor	**Coef**	**Stdev**	**t-ratio**	**p**
Constant	-0.11254	0.06400	-1.76	0.080
SPX	1.11269	0.06391	17.41	0.000

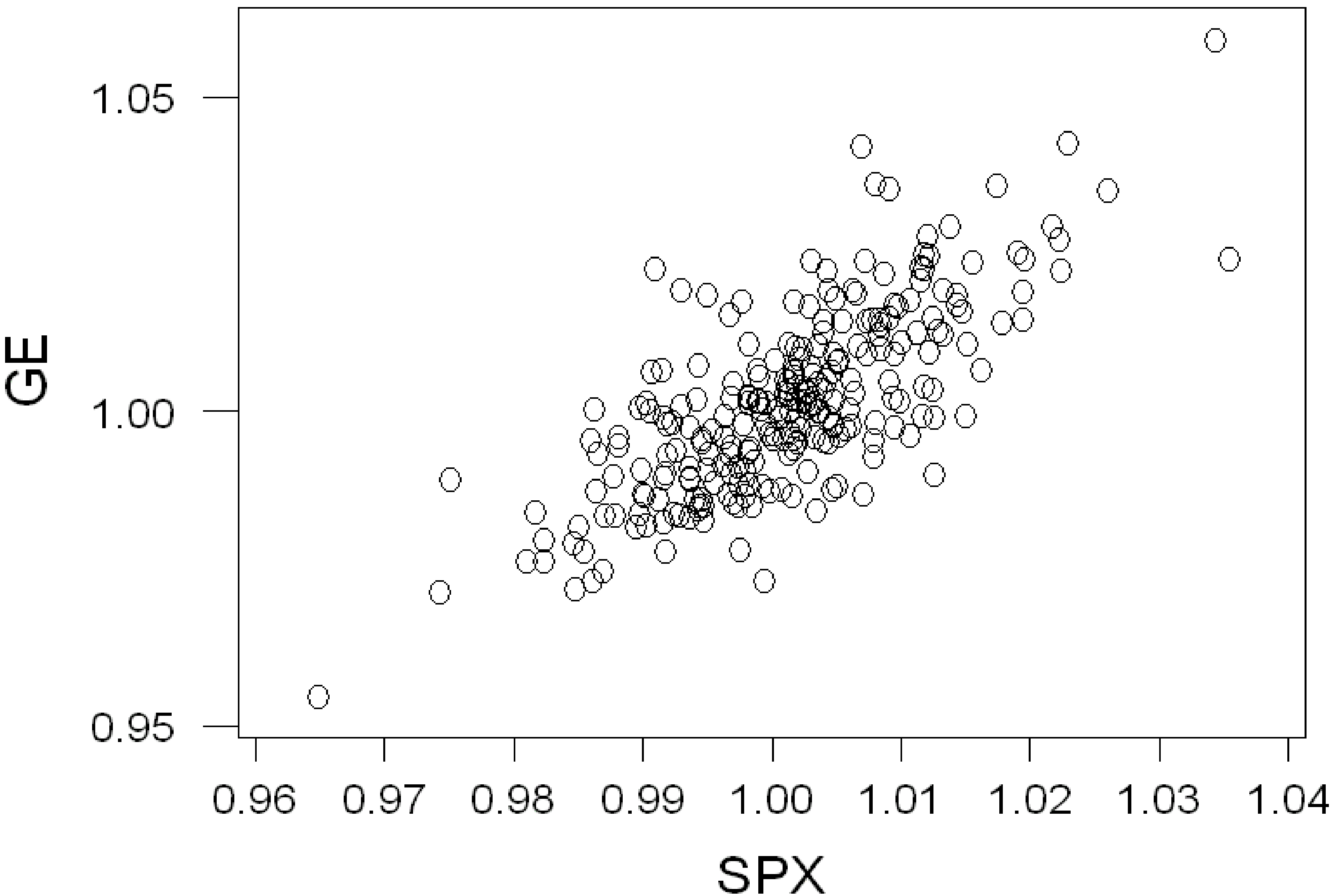
$\sigma = 0.009888$ R-sq = 54.9%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	0.029638	0.029638	303.11	0.000
Error	249	0.024348	0.000098		
Total	250	0.053986			

Large t-ratio(>2.306) are statistically significant @ 5% level

Small p value's (<0.10) are statistically significant.


GE
1.05
1.00
0.95
0.96
0.97
0.98
0.99
1.00
1.01
1.02
1.03
1.04
SPX

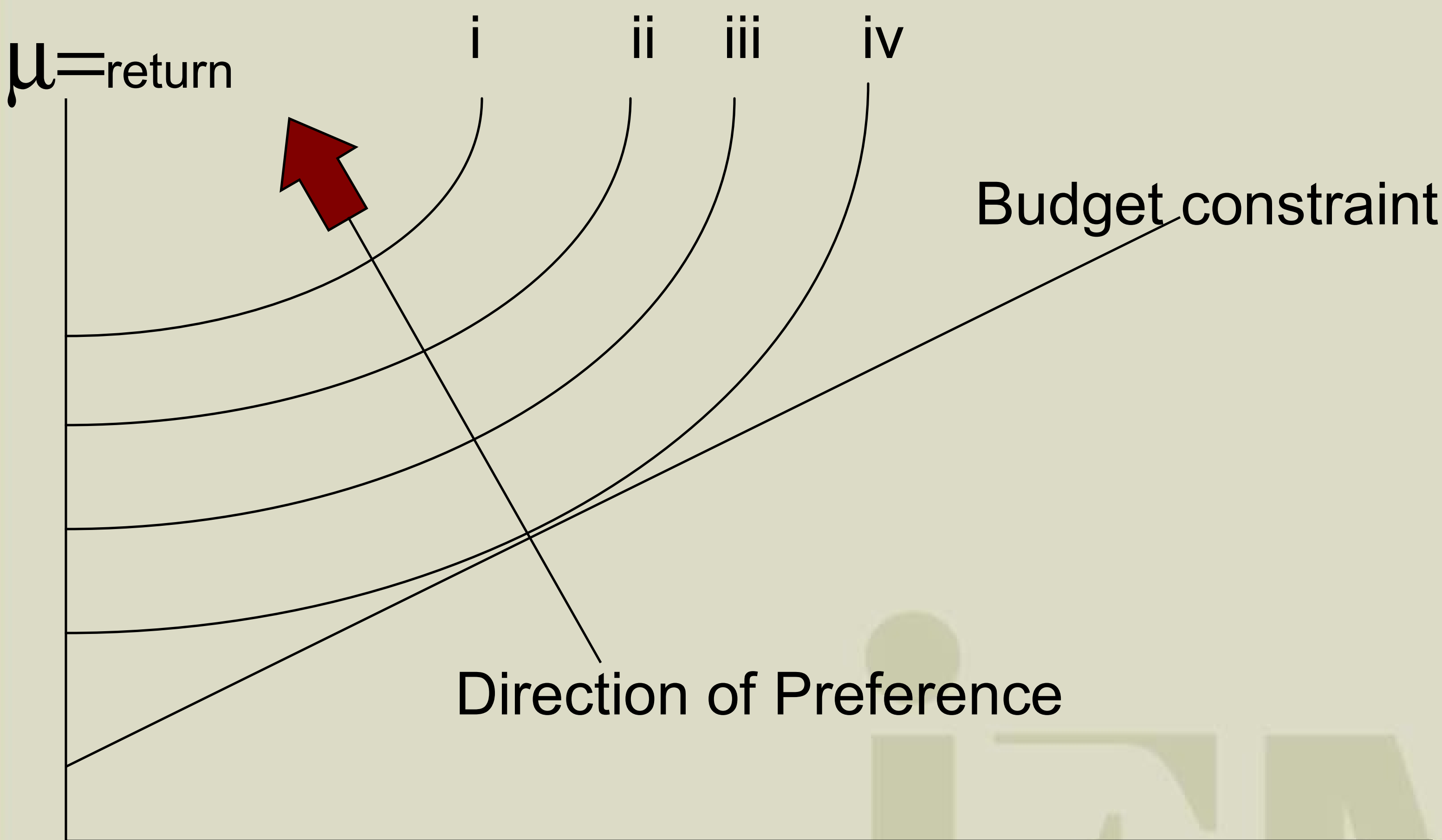
Tests of the Market Model

1. The residual can be used to judge whether the market is efficient in adjusting prices for company specific information.

   i.e. Is $\Sigma e_i = 0.0$?

2. Event studies - How fast is the market in adjusting to new information? If markets are efficient, you shouldn't see positive or negative drift in e_i -that is, the residuals should 'look' random.

Utility = fn(μ, σ)= return & risk



where: μ= return, σ = risk

Risk for an individual Stock

$$\sigma\,(R_i) = \left(\frac{1}{n-1}\ \Sigma\,(R_i - \bar{R})^2\,P_i\right)^{1/2}$$

Where P_i = probability of event R_i occuring

Risk & Return for a Portfolio

$$E(R_{portfolio}) = \sum_{i=1}^{n} w_i \, E(R_i)$$

That is, the expected return on the portfolio is just the weighted average expected return of the component stocks.

However, the portfolio risk ($\sigma_{portfolio}$) is <u>not the weighted average of the component stock risk factors</u>.

$$\sigma_{portfolio} \neq \sum_{i=1}^{n} (w_i \, \sigma_i)$$

Why? Covariance!

$$Cov\,(R_1,R_2) = \sum_{i=1}^{n} P_i[R_{1i} - E(R_1)][R_{2i} - E(R_2)]$$

$$= \sum_x \sum_y (x-\mu_x)(y-\mu_y)P_{xy}(x,y)$$

$$= [\sum_x \sum_y xy\, P_{xy}(x,y)] - \mu_x\, \mu_y$$

Portfolio risk equation:

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\, cov(R_1, R_2)$$

Three Stock Portfolio

			Price Ret.Vol.	
n=250	Last Price[1]	$E(Ret._i)$	σ_{daily}	σ_{annual}
GE:	22.5	11.5%	0.0147	0.2324
IBM:	77.45	8.75%	0.0138	0.2182
SBC:	23.35	12%	0.0196	0.3099

[1] Feb.22,2005

Pairwise Covariance GE, IBM, SBC
Feb 15,2004 – Feb 15, 2005

	GE	IBM	SBC
GE	0.00021594		
IBM	0.00010997	0.00018990	
SBC	0.00012369	0.00010215	0.00038257

$\sigma^2_{return,i}$ along diagonal

Cov(GE, IBM) off diagonal

$-\infty < cov(GE, IBM) < +\infty$

MINIMIZE the Portfolio Variance

$$\sigma^2_{port.} = (.21594 \times GE^2) + (.1899 \times IBM^2) + (.38257 \times SBC^2) +$$
$$2 \times 0.10997 \times GE \times IBM +$$
$$2 \times 0.12369 \times GE \times SBC +$$
$$2 \times 0.10215 \times IBM \times SBC$$

where: <u>Variances</u>

$\sigma^2_{GE} = 0.21594^*$

$\sigma^2_{IBM} = 0.1899^*$

$\sigma^2_{SBC} = 0.38257^*$

and: <u>pairwise covariances</u>

$\sigma_{GE,IBM} = 0.10997^*$

$\sigma_{GE,SBC} = 0.12369^*$

$\sigma_{IBM,SBC} = 0.10215^*$

GE = fraction of portfolio invested in GE
IBM = fraction of portfolio invested in IBM
SBC = fraction of portfolio invested in SBC

* for actual figures divide by 10,000.

Non-Linear Optimization

MODEL:

MINIMIZE PORTFOLIO VARIANCE

1) MIN= 0.21594 X GE^2+0.1899 X IBM^2+0.38257 X SBC^2) +
2 X 0.10997X GE X IBM+
2 X 0.12369 X GE X SBC+
2 X 0.10215 X IBM X SBC;

SUBJECT TO:

2) GE + IBM +SBC = 1.0 ; BEING FULLY INVESTED

3) 1.115 X GE +1.0875 X IBM + 1.12 X SBC > 1.1;MIN. Expected.Return at least =10%

END

BOUNDRY CONSTRAINTS.

SLB	GE	.000000
SUB	GE	.400000
SLB	IBM	.000000
SUB	IBM	.400000
SLB	SBC	.000000
SUB	SBC	.400000

lower boundry set at zero to insure no short position; upper boundry set at 40% allocation for any one stock.

What's Best: solution

WBMIN f_x =F3*C3^2+G4*C4^2+H5*C5^2+2*F4*C3*C4+2*F5*C3*C5+2*G5*C4*C5

	A	B	C	D	E	F	G	H
1							CoVar Matrix	
2			% invested	Exp(Ret.)		GE	IBM	SBC
3		GE	40.00%	11.50%		0.21594000		
4		IBM	40.00%	8.75%		0.10997000	0.18990000	
5		SBC	20.00%	12.00%		0.12369000	0.10215000	0.38257000
6								
7		Portfolio Return	10.500%					
8								
9	Objective: Min. Portfolio Risk		0.151562	=Variance				
10			38.9%	=Std.Dev.				
11	Subject to:						Ignoring CoVar	
12	invest it all	1.0	=	1		GE's risk contrib.@ 40%	18.6%	
13	not short GE	40.00%	>=	0		IBM risk contrib .@ 40%	17.4%	
14	not short IBM	40.00%	>=	0		SBC risk contrib.@ 20%	12.4%	
15	not short SBC	20.00%	>=	0			48.4%	Proportional
16	max 40% GE	40.00%	=<=	40%				weighted
17	max 40% IBM	40.00%	=<=	40%				standard dev.
18	max 40% SBC	20.00%	<=	40%				
19	portfolio return	10.500%	>=	10%				

Diversification and risk

- With perfect correlation, +1.0, the risk of the portfolio IS the weighted average of individual stock σ's.
- With perfect negative correlation, -1.0, it is possible to combine stocks to eliminate ALL risk.
- The risk of a portfolio is determined *PRIMARILY* by the COV and not the σ^2. However, an individual stock's risk is measured by σ^2.

Efficient Portfolio's

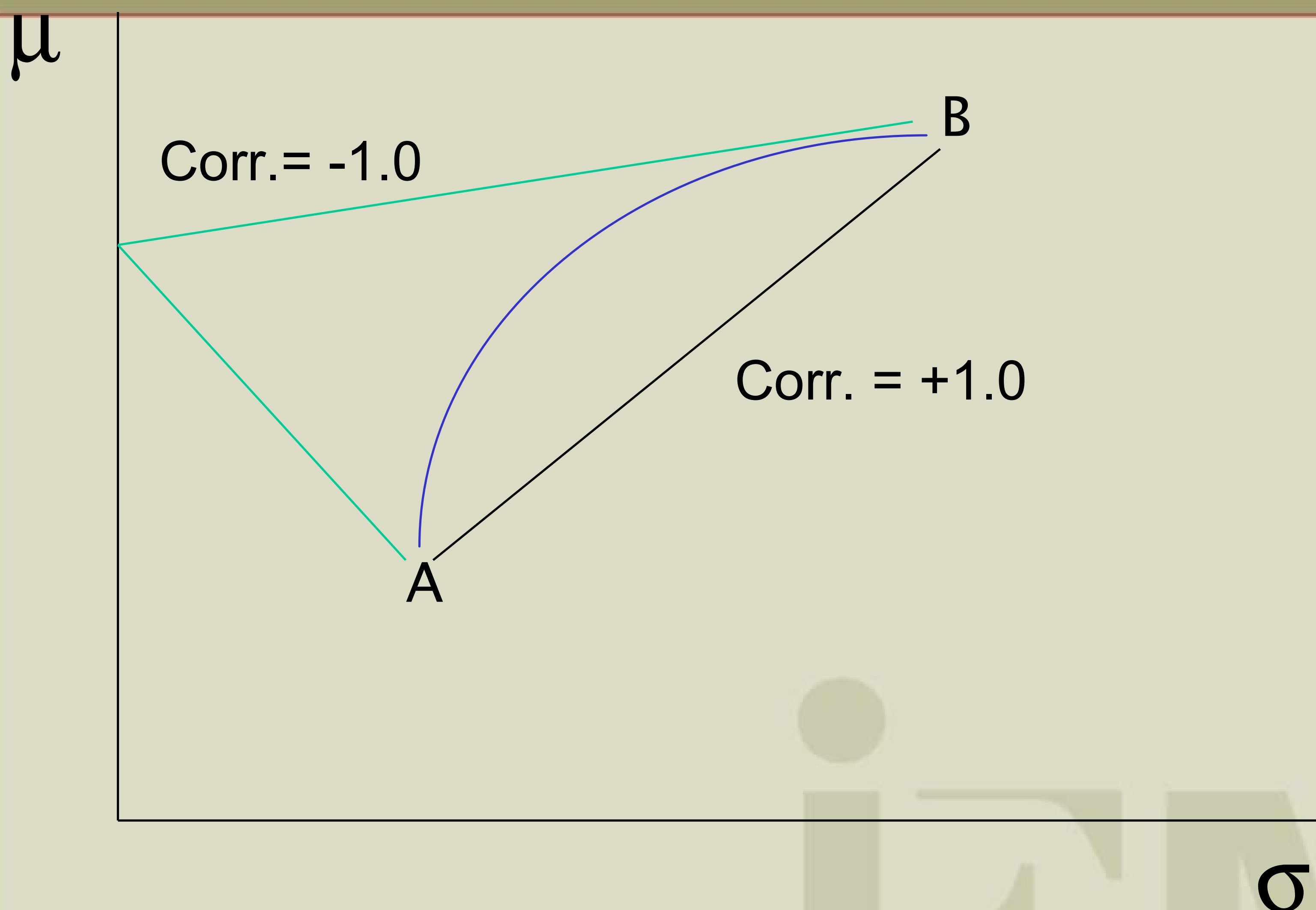
Maximize E(Return) for a given amount of σ

or

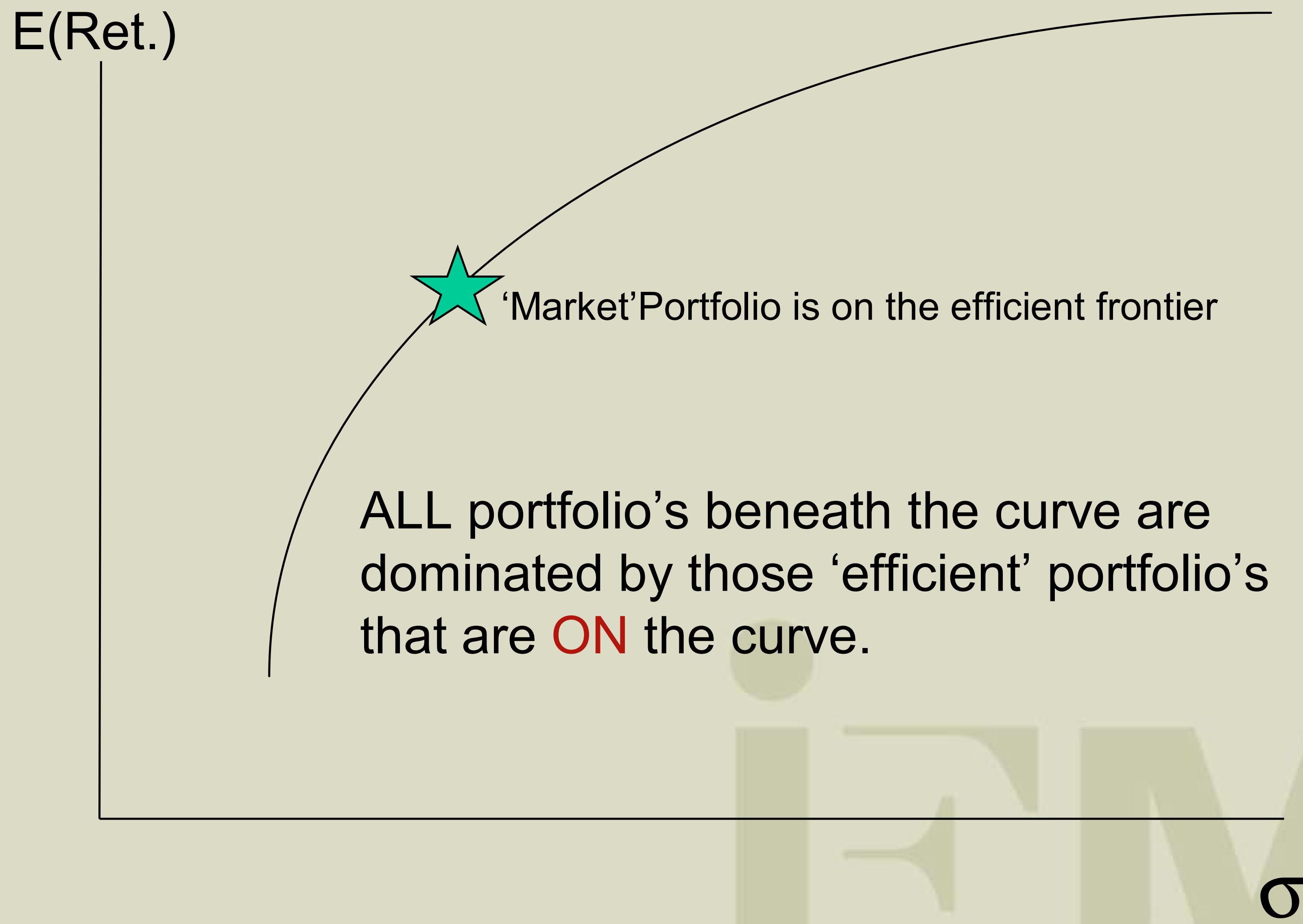
Minimize σ for a given E(Return)

Problem: Markowitz diversification is only feasible for perhaps 0-30 stocks because it requires all pairwise covariances.

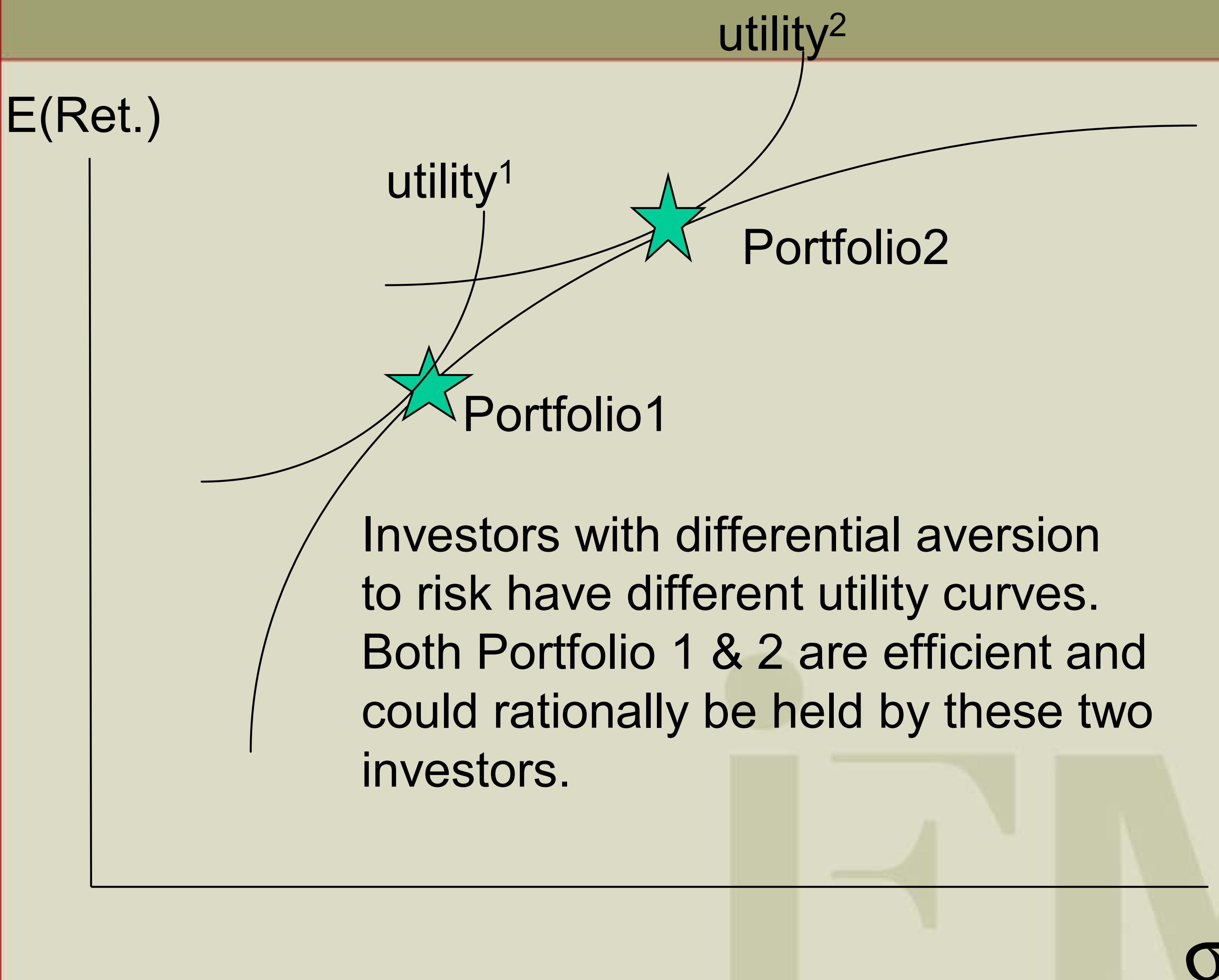
Combining Assets into Portfolio's



Again, because the market effect is so strong, negatively correlated stocks are extremely difficult to find.

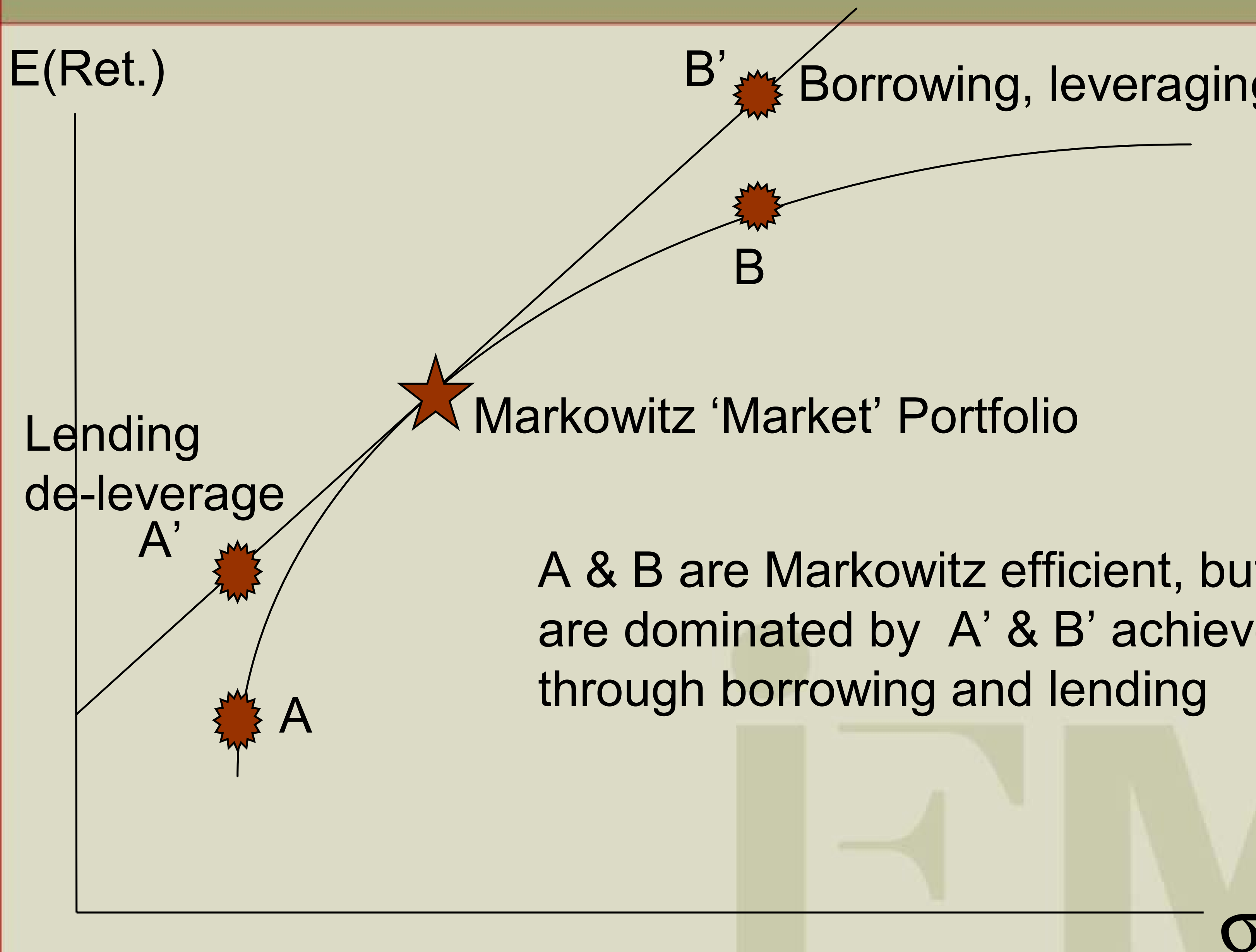


ALL portfolio’s beneath the curve are dominated by those ‘efficient’ portfolio’s that are ON the curve.



Investors with differential aversion to risk have different utility curves. Both Portfolio 1 & 2 are efficient and could rationally be held by these two investors.

Capital Market Line 'CML'

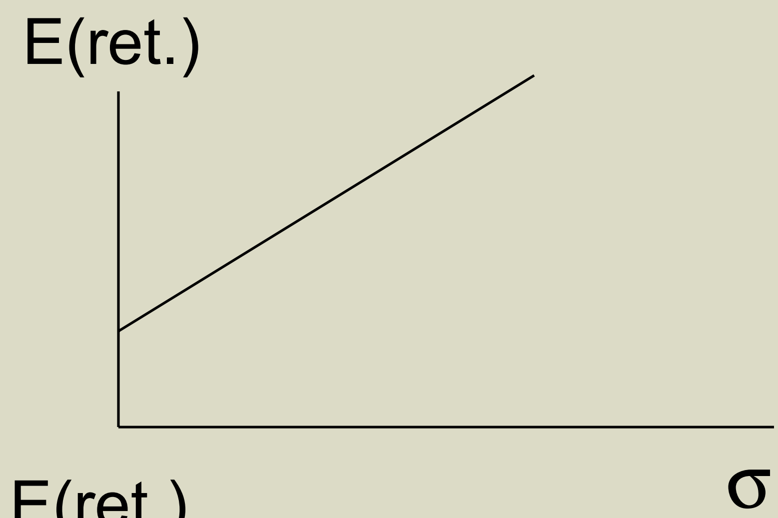


A & B are Markowitz efficient, but are dominated by A' & B' achieved through borrowing and lending

Equilibrium Models

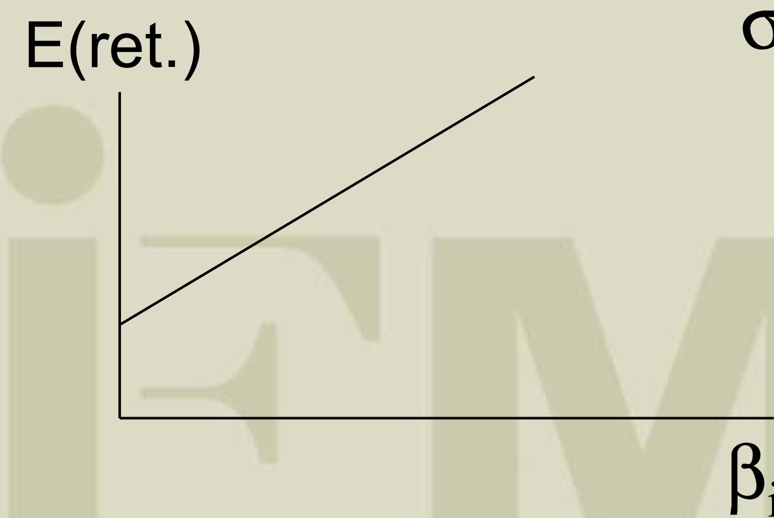
Capital Market Line:

$$E(R_P) = R_{free} + \sigma_{MP}\left[\frac{E(R_m) - R_{free}}{\sigma_{MP}}\right]$$



Security Market Line:

$$E(R_i) = R_{free} + \beta_{i,m}[E(R_m) - R_{free}]$$



CML vs. SML

Capital Market Line: a *very specific* formula useful only for efficient portfolio's; risk unit is σ.

Security Market Line: Useful for both efficient and inefficient portfolio's; risk unit is β.

where:

σ_{MP} = Standard deviation of Market Portfolio

$$\beta_{i,m} = \frac{S_{xy}}{S_{xx}} \quad \begin{array}{l} = \\ = \end{array} \frac{\Sigma x_i y_i - (\Sigma x_i)(\Sigma y_i)/n}{\Sigma x_i^2 - (\Sigma x_i)^2/n}$$

Where x = SP500
y = stock_i

CAPM {Sharpe-Lintner}

Assumptions:

1. Frictionless capital markets-No transaction costs, No taxes,etc.
2. Homogenous expectations -everyone interprets information alike.
3. Information is free and available to everyone.
4. You may borrow/lend at R_{free} rate in unlimited quantity.
5. Maximize E(Utility)
6. One period world -Don’t use CAPM over multiple periods.

What is CAPM for?

1. Determining the appropriate rate of return for a given risk level.

2. Developing the ideal investment strategy.

CAPM

$$R_i = R_{free} + \overbrace{\beta_i}^{\text{relative risk of asset}_i \text{ to the market}} \underbrace{[\, R_{mkt} - R_{free} \,]}_{\text{market price of risk}}$$

Says: The required rate of return is equal to the risk free rate PLUS the market price of risk adjusted for the relative risk of the asset.

Evidence

Research suggests that 70% of mutual fund managers <u>*UNDERPERFORM*</u> the S&P 500 index.

Frequently, they underperform by an amount that is approximately equal to their expense ratio 1.0-1½ %

Strategy

If you desire β =1.2; Buy index on margin (borrow)

If you desire β =0.8; Buy index & buy T-bill (lend)

Measuring Performance

Sharpe Perfomance Index: $$S_p = \frac{\overline{R}_p - \overline{R}_{free}}{\sigma_p}$$

Treynor Index: $$T_p = \frac{\overline{R}_p - \overline{R}_{free}}{\beta_p}$$

Jensen~alpha: $$R_p - R_f = \alpha_{p,m} + \beta_{p,m}[E(R_m - R_{free})] + \varepsilon_i$$

Risk Adjusted Returns

	Sharpe	Treynor
Mgr. A : Portfolio Return = 40% σ_A = 25%, r_{free} = 3.00% β_A = 1.6	1.48	0.231
Mgr. B: Portfolio Return = 25% σ_B = 10%, r_{free} = 3.00% β_B = 0.4	2.20	0.925

Which portfolio do you prefer?

The larger the statistic, the more attractive the risk adjusted return.

Jensen's~alpha {adjusted CAPM}

$E(\varepsilon_i)=0.0$

$$\underbrace{R_p - R_{free}}_{\text{Premium earned}} = \alpha_{p,m} + \underbrace{\beta_{p,m}}_{\text{Adjusted for Portfolio risk}} [\underbrace{E(R_m - R_{free})}_{\text{Market price of risk 'Risk premium'}}] + \varepsilon_i$$

Alpha Expectations

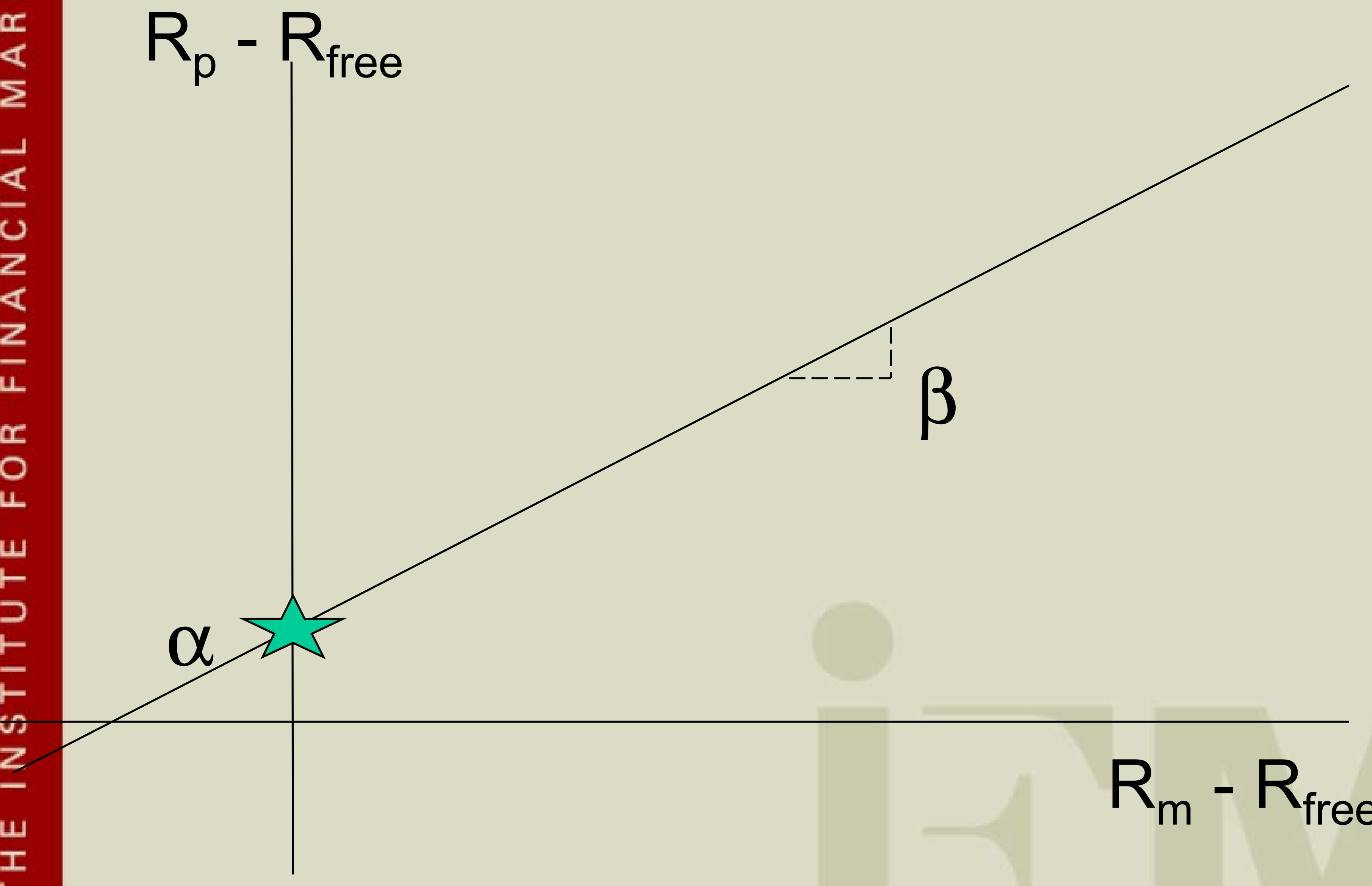
$$E(\alpha_{p,m}) = 0.0$$

if $\alpha_{p,m} > 0.0$ superior results

if $\alpha_{p,m} < 0.0$ inferior results

Positive alpha- the rate of return an investor is able to earn above an unmanaged portfolio with the same risk.

Jensen's Alpha



Jensen-Alpha & Mutual Funds

A strong form test to evaluate whether professional managers offer any advantage over buying the index.

Results: $\bar{\alpha} = -1.1\%$, and $\bar{\beta} = 0.84$

Ignoring transaction costs, $\alpha = -0.4\%$

There is no evidence that mutual fund managers, as a group, can beat the market.

When will CAPM Fail?

When $\hat{\beta}_{i,m}$ is not a good estimate of $\beta_{i,m}$

When $\overline{R}_m$ is not a good estimate of $E(R_m)$

When $E(R_i) \neq R_{free} + \beta_{i,m} [E(R_m) - R_{free}]$

Fama & MacBeth

(a rigorous test of CAPM)

Postulated that the return to $stock_i$ is potentially made up of <u>four</u> components

1. Risk free rate
2. Systematic risk
3. Non-linear effect
4. Non-systematic risk

Parameters designed to capture:

$$R_p = \overbrace{\alpha_{0,t}}^{R_{free}} + \underbrace{\alpha_{1,t}\,(\beta_{p,t-1})}_{\text{Systematic risk}} + \alpha_{2,t}\,\overbrace{(\beta_{p,t-1})^2}^{\text{Non-linear effect}} + \underbrace{\alpha_{3,t}(\sigma_{p,t-1})}_{\text{Firm specific risk}} + \varepsilon_i$$

Fama/MacBeth Results

$\alpha_{0,t}$: Close to 30 day T-bill rate; but not statistically different from zero. Supports CAPM.

$\alpha_{1,t}$: Found positive and significant. Captures the risk premium exactly predicted by CAPM.

$\alpha_{2,t}$: Not statistically different from zero.

$\alpha_{3,t}$: Not statistically different from zero.

Conclusion: Supports CAPM. It is possible to have non-linear risk/return relationship in short-run; however, **in the long run risk/return ARE LINEAR**.

Banz

Tried to incorporate the SIZE effect into the model.
"Size" refers to the market capital of the firm relative to the total market capital.

$$R_i = \alpha_0 + \alpha_1\beta_1 + \alpha_2 \left[\frac{\phi_i - \phi_m}{\phi_m}\right] + \varepsilon_i$$

Where:

ϕ_i = market capital of firm

ϕ_m = market capital of the market

and, Expectation of $\Sigma\, \varepsilon_i = 0.0$

Banz Results

α_0 is higher than R_{free} , supports Black's Z portfolio
α_1 is positive and significant - supports CAPM
α_2 is negative and very significant -this means small firms outperform the market.

Conclusion

CAPM is mis-specified because it doesn't capture the size effect. Also found that Beta is *not the only* measure of systematic risk.

Thank You

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